Beyond the Standard Model at LEP

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Beyond the Standard Model at LEP

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LEP data constrain severely many proposed extensions of the Standard Model. These include: massive neutrinos, which are now largely excluded as candidates for the dark matter of the Universe; supersymmetric particles, the lightest of which would still constitute detectable dark matter; technicolour, of which many favoured versions are now excluded by precision electroweak measurements; and grand unified theories, of which LEP data favour supersymmetric versions.

1. The Standard Model and its problems

There are basically two ways to look for physics beyond the Standard Model with LEP or any other accelerator: look directly for some new particle or interaction, or make precision measurements and looking indirectly at effects due to virtual particles too heavy to be produced directly. LEP is ideally suited for both strategies: because of its clean experimental conditions it is able to explore completely the spectrum of particles that are kinematically accessible, and the high event rate at the Z^0 peak opens the way to high-precision measurements. Before following up on either of these strategies, let us first recall some essential features of the Standard Model and problems that it leaves open.

The Standard Model has 20 free parameters, which are the following: three gauge couplings and two non-perturbative vacuum angle parameters, nine fermion masses, and two parameters to characterize the Higgs sector that can be taken as the quartic coupling λ and a negative mass-squared parameter $-\mu^2$, or equivalently as m_H and m_W. The open problems can be classified into areas associated respectively with these gauge, flavour and mass sectors. In the gauge sector, one would hope to unify the different $SU(3)$, $SU(2)$ and $U(1)$ gauge group factors into a simple group G with a single coupling and a non-perturbative vacuum parameter, in a grand unified theory (GUT) (Pati & Salam 1973; Georgi & Glashow 1974). In the flavour sector, it has often been suggested that quarks and leptons might be composite. However, if one respects all the theoretical consistency conditions, composite models turn out to more complicated than the 'Mendeleev Table' they seek to simplify, so I find them uninteresting and will not discuss them further here. In the mass sector, it remains to verify the Higgs mechanism, and theorists are concerned about the quadratically divergent radiative corrections to the elementary Higgs potential parameter μ, or equivalently m_H and m_W. To avoid these radiative corrections dragging m_W off to $m_P \approx 10^{19}$ GeV, many theorists either invoke supersymmetry to cancel the quadratic divergences (Maiani 1980; Witten 1981; Dimopoulos & Georgi 1981; Sakai 1982) or else postulate that the Higgs boson is not elementary, but composite. We see later how these ideas fare with LEP data. Eventually, theorists hope to find a theory of everything (TOE) that solves all these problems and also encompasses quantum gravity. String theory (Green *et al.* 1986) is the only candidate TOE, and we see in the

last section how it accommodates grand unification and supersymmetry, and try to find some characteristic experimental predictions from string.

2. Implications of some direct searches

The many LEP searches for new particles are reviewed at this meeting (Green 1991), and here I simply focus on the implications of some of them, in particular as they concern supersymmetric models. The first search that I consider is that for invisible neutral particles, as summarized by the LEP measurement (Stone 1991) of the number of apparent neutrino species:

$$N_\nu = 3.01 \pm 0.05. \tag{1}$$

In addition to determining the number of conventional light neutrinos to be three, this measurement also constrains possible other neutral particles that are either massive (Griest & Silk 1990) and/or weakly coupled to the Z^0 (Ellis *et al.* 1990*d*). For example, if we interpret the measurement (1) as implying that $N_\nu < 3.10$, we can infer the following lower limits on any massive Dirac neutrino:

$$m_{\nu_D} > 45 \, \text{GeV} \tag{2a}$$

on any massive Majorana neutrino:

$$m_{\nu_M} > 40 \, \text{GeV} \tag{2b}$$

and on sneutrinos (assuming three degenerate flavours):

$$m_{\tilde\nu} > 43 \, \text{GeV}. \tag{2c}$$

These lower limits effectively exclude massive neutrinos as candidates for the dark matter of the Universe. According to standard Big Bang cosmology, the relic energy density of massive neutrinos is proportional to m_ν for $m_\nu \lesssim 1 \, \text{MeV}$, and becomes unacceptably high when $m_\nu \gtrsim 100 \, \text{eV}$. The neutrino energy for more massive neutrinos falls again, suppressed by a Boltzmann factor *ca.* $\text{e}^{-m_\nu/T}$, becoming cosmologically acceptable when $m_\nu \gtrsim 1\text{–}10 \, \text{GeV}$. The relic energy density is very small for neutrinos (or sneutrinos) satisfying the mass constraints (2) effectively ruling out massive neutrinos or stable sneutrinos as dark matter, unless they weigh much more than $\frac{1}{2}m_Z$.

Another candidate for dark matter is provided by supersymmetry. According to this theory, all the known particles are accompanied by supersymmetric partners with identical internal quantum numbers (electric charge, strong coupling, etc), but with spins differing by half a unit. Thus fermions (e.g. quarks q and leptons l) are spartnered by spin-0 bosons (squarks q̃ and sleptons l̃), and bosons (e.g. the photon γ, the $W^\pm$, etc.) by spin-$\frac{1}{2}$ fermions (the photino $\tilde\gamma$, the charged wino $\tilde W^\pm$, etc.). The gauge interactions in the minimal supersymmetric extension of the Standard Model are identical with those in the Standard Model (Haber & Kane 1985; Nilles 1984), as are the Yukawa iterations. Between them, they determine the effective Higgs potential in the limit of exact supersymmetry. In point of fact, we know that supersymmetry must be broken, since $m_{\tilde e} \neq m_e$, $m_{\tilde\gamma} \neq m_\gamma$, etc. This necessary supersymmetry breaking is usually parametrized by introducing scalar and gaugino mass parameters:

$$\mathscr{L}_{\text{SUSYX}} \ni -\sum_i m_{O_i}^2 |\phi^i|^2 - \sum_{a=1}^{3} M_a \tilde V_a \tilde V_a + \dots. \tag{3}$$
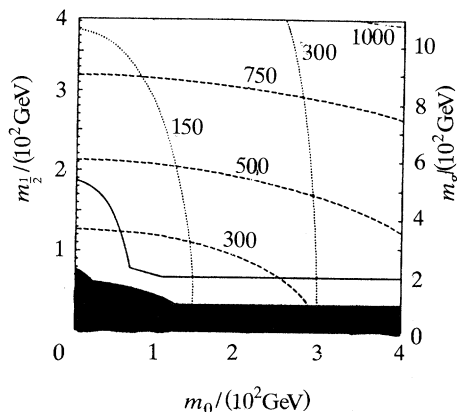
Figure 1. Region of the supersymmetry-breaking parameters m_0 and $m_{\frac{1}{2}}$ excluded by LEP and CDF data for $\tan\beta \equiv v_2/v_1 = 2$ (Ridolfi *et al.* 1990).

Ultimately, it is expected that these parameters are generated by some high-energy supergravity or superstring theory, in which the scalar (gaugino) masses are likely to be universal at some large energy scale $Q \approx m_P$:

$$M_{O_i}|_{Q \approx m_P} \equiv m_{\frac{1}{2}}. \tag{4}$$

These values would be renormalized in the effective low-energy theory, much like the gauge couplings in GUTs. Typical calculations yield (Lahanas & Nanopoulos 1987)

$$m_{O_i}^2|_{Q \approx m_W} \approx m_0^2 + C_i\, m_{\frac{1}{2}}^2 : C_{\tilde{q}} \approx 7,\, C_{\tilde{l}_L} \approx \tfrac{1}{2},\, C_{\tilde{l}_R} \approx 0.15 \tag{5a}$$

and

$$M_a|_{Q \approx m_W} \approx (\alpha_a/\alpha_{GUT})\, m_{\frac{1}{2}} : m_{\tilde{g}} \approx 3m_{\frac{1}{2}},\, m_{\tilde{\gamma}} \approx \tfrac{1}{2}m_{\frac{1}{2}}. \tag{5b}$$

The estimates (5) can then be used to compare the constraints on the masses of super-symmetric particles provided by different experiments, as seen in figure 1 (Ridolfi *et al.* 1990). We see that although the Fermilab Tevatron collider has a larger mass reach, LEP compensates by being able to search for electroweakly interacting sparticles such as sleptons and winos.

In the minimal supersymmetric extension of the Standard Model, the charged winos and higgsino have squared-mass mixing given by the following matrix:

$$M_{\tilde{C}}^2 = \begin{pmatrix} M_2^2 + g_2^2\, v_1^2 & -M_2\, g_2\, v_2 + \mu g_2\, v_1 \\ -M_2\, g_2\, v_2 + \mu g_2\, v_1 & \mu^2 + g_2^2\, v_2^2 \end{pmatrix}, \tag{6}$$

where μ is an $H_1 H_2$ mixing term in the superpotential. The four neutralinos $\tilde{W}^3$, $\tilde{B}$, $\tilde{H}_2^0$, $\tilde{H}_1^0$ have the following mass mixing matrix:

$$M_{\tilde{N}} = \begin{pmatrix} M_2 & 0 & \frac{1}{\sqrt{2}} - g_2\, v_2 & \frac{1}{\sqrt{2}} - g_2\, v_1 \\ 0 & M_1 & \frac{1}{\sqrt{2}} g'\, v_2 & \frac{1}{\sqrt{2}} g'\, v_1 \\ \frac{1}{\sqrt{2}} - g_2\, v_2 & \frac{1}{\sqrt{2}} g'\, v_2 & 0 & \mu \\ \frac{1}{\sqrt{2}} g_2\, v_1 & \frac{1}{\sqrt{2}} - g'\, v_1 & \mu & 0 \end{pmatrix}. \tag{7}$$

The lightest neutralino, χ, is in general a complicated mixture, but becomes relatively simple in the following limits:

$$M_2, M_1 \to 0, \quad \chi \approx \tilde{\gamma}, \quad m_\chi \approx \tfrac{8}{3} g'^2 M_2/(g_2^2 + g'^2), \tag{8a}$$
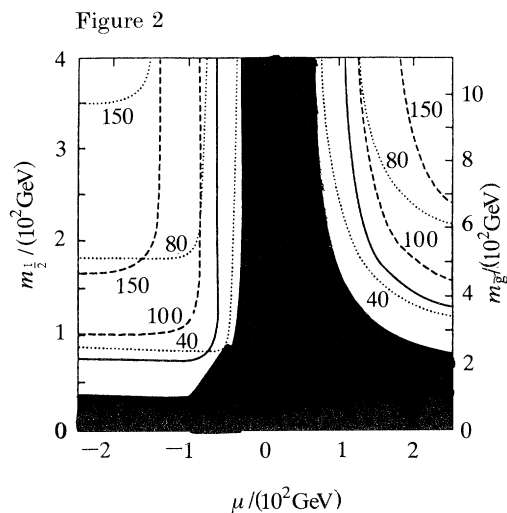
$$\mu \to 0, \quad \chi \approx \tilde{H} \tag{8b}$$

Figure 2

Figure 3

Figure 2. Region of the H_1, H_2 mixing parameter μ and the supersymmetry-breaking parameter $m_{\frac{1}{2}}$ excluded by LEP and CDF data for $\tan\beta = 2$ (shaded area) (Ridolfi *et al.* 1990).
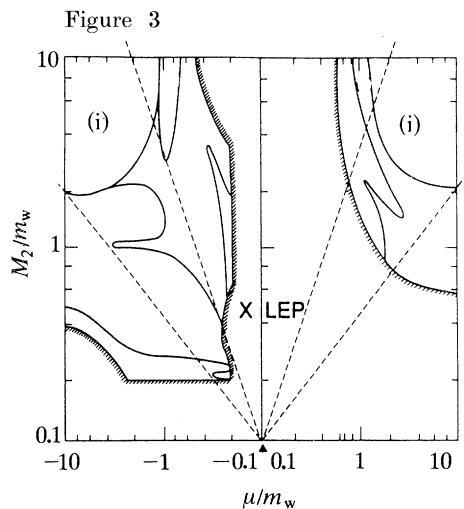
Figure 3. Regions of the $(\mu, m_{\frac{1}{2}})$ plane in which the lightest supersymmetric particle (LSP) may have the closure density $\Omega = 1$ for some choice of the parameters ($\tan\beta$, m_A, $m_{\tilde{q}}$) (Ellis *et al.* 1990d); region (i), $m_{LSP} > m_W$.

either of which could constitute the dark matter in the Universe (Ellis *et al.* 1984).

Constraints on the parameters of the gaugino/higgsino mass matrices are provided primarily by LEP: $m_{\tilde{W}^\pm} > 45\,\mathrm{GeV}$, σ_{peak}, and upper limits on $\Delta\Gamma_Z^{\mathrm{total}}$, ΔN_ν, $Z^0 \to \chi(\chi' \to \chi + \mathrm{visible})$, and also by hadron–hadron colliders: $m_{\tilde{g}} = (\alpha_3/\alpha_2)M_2$, $\sigma B(W \to \mathrm{e}\nu/\sigma B(Z \to \mathrm{e}^+\mathrm{e}^-)$. Figure 2 shows a compilation of these constraints (Ellis *et al.* 1990c; Ridolfi *et al.* 1990). These are still compatible with χ constituting the dark matter for some possible value of $\tan\beta \equiv v_2/v_1$, the pseudoscalar Higgs m_A, and the squark mass $m_{\tilde{q}}$. Figure 3 shows the regions of the (μ, M_2) plane where $\Omega = \rho_\chi/\rho_{\mathrm{critical}} = 1$ for some choice of these parameters (Ellis *et al.* 1990d). The χ could be either an approximate $\tilde{\gamma}$, $\tilde{H}$ or $\tilde{Z}$, albeit somewhat heavier than had often been assumed in pre-LEP studies. Detailed implications for dark matter detection, given the LEP mass limits, remain to be worked out.

In the minimal supersymmetric extension of the Standard Model with two Higgs doublets there are five physical Higgs bosons: two scalars h, H, one pseudoscalar A and two charged bosons $H^\pm$. At the tree level, their masses obey the following relations (Haber & Kane 1985):

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2, \quad m_A^2 = m_{H^\pm}^2 - m_{W^\pm}^2, \tag{9}$$

$$m_{h,H}^2 = \tfrac{1}{2}[m_A^2 + m_Z^2 \pm \sqrt{[(m_A^2 + m_Z^2) - 4m_Z^2 m_A^2 \cos^2 2\beta]}]. \tag{10}$$

These guarantee that the lightest neutral Higgs h weighs less than m_Z, while the heavier scalar H must weigh more than m_Z. All the Higgs masses and coupling are determined in terms of two parameters that may be taken as (m_h, m_A) or (m_A, $\tan\beta$). If one accepted the tree-level mass formulae, a large range of the (m_h, m_A) plane would apparently be excluded by present LEP data, and the range accessible to experiments at LEP II would extend almost as far as the theoretical limit $m_h \lesssim m_Z$.

Unfortunately, there are large radiative corrections to the masses of the super-symmetric Higgs bosons that alter this picture drastically (Ellis *et al.* 1991a; Haber
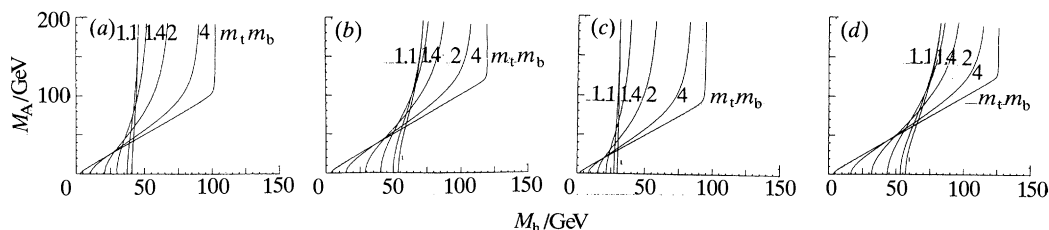
Figure 4. The (m_h, m_A) plane after including radiative corrections to the masses calculated using the following values of m_t, $m_{\tilde{q}}$, μ and the soft supersymmetry breaking parameters A_t, A_b (Ellis *et al.* 1991b): (a) $m_t = 120\,\text{GeV}$, $m_{\tilde{q}} = 1\,\text{TeV}$, $A_t = A_b = \mu = 0$; (b) $m_t = 160\,\text{GeV}$, $m_{\tilde{q}} = 1\,\text{TeV}$, $A_t = A_b = \mu = 0$; (c) $m_t = 120\,\text{GeV}$, $m_{\tilde{q}} = 150\,\text{GeV}$, $A_t = A_b = \mu = 100\,\text{GeV}$; (d) $m_t = 160\,\text{GeV}$, $m_{\tilde{q}} = 1\,\text{TeV}$, $A_t = A_b = 1\,\text{TeV}$, $\mu = 200\,\text{GeV}$.
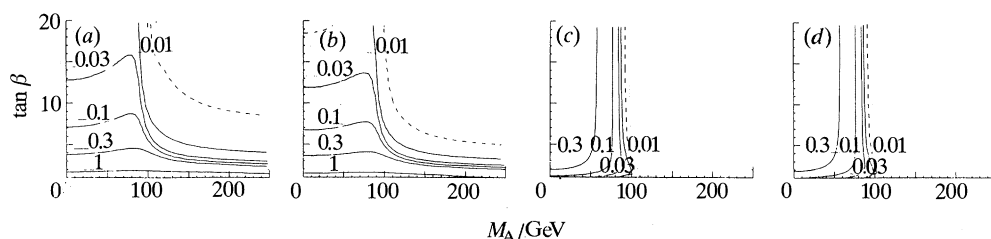

Figure 5. Region of the $(m_A, \tan\beta)$ plane accessible to LEP II after including radiative corrections to the masses (Ellis *et al.* 1991b). (a), (b) $\sigma\,(\text{e}^+\text{e}^- \to \text{hZ}^0)$; (c), (d) $\sigma\,(\text{e}^+\text{e}^- \to \text{hA})$. (a), (c) $A_t = A_b = \mu = 0$; (b), (d) $A_t = A_b = 1\,\text{TeV}$, $\mu = 200\,\text{GeV}$. $----$, $\sqrt{s} = 190\,\text{GeV}$.

& Hempfling 1991). In particular, they increase m_h, which could well exceed m_Z. The physical Higgs masses are given to a good approximation by the full one-loop effective potential:

$$m_{\text{phys}}^2 \approx (\partial^2 V / \partial \phi^2)|_{\phi = v} : V = V_0(Q) + V_1(Q), \tag{11}$$

where Q is an arbitrary renormalization scale and

$$V_1(Q) = (1/64\pi^2)\,\text{Str}\,\mathcal{M}^4\,(\ln\,(\mathcal{M}^2/Q^2) - \tfrac{3}{2}). \tag{12}$$

The dominant contribution is that due to the t quark, which is proportional to λ_t^2 and hence m_t^2, and would be quadratically divergent in the absence of supersymmetry. Hence the radiative correction is proportional to m_t^4 for a fixed ratio of $m_{\tilde{t}}/m_t$:

$$\frac{\partial V_1}{\partial \phi^2}\bigg|_{\phi = v} = \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\ln\frac{m_{\tilde{t}}^2}{m_t^2}\bigg|_{\phi = v}. \tag{13}$$

These corrections can range up to *ca.* 100 GeV. They give a large value of m_h even in the limit $\tan\beta \to 1$, where the tree level formulae would indicate that $m_h \to 0$. Figure 4 shows areas of the (m_h, m_A) plane that are physically allowed for some values of m_t and $m_{\tilde{q}}$. We see that m_h is bounded away from zero, that $m_h > m_Z$ is possible, and also that $m_h > m_A$ is possible (Ellis *et al.* 1991a, b). These results affect the interpretation of present LEP searches for supersymmetric Higgses, which can no longer be used to infer a lower limit on $\tan\beta$, and cloud the prospects for detecting supersymmetric Higgses even at LEP II. Figure 5 shows regions of parameter space that remain accessible to LEP II after radiative corrections to the Higgs masses are included (Ellis *et al.* 1991b).

3. Electroweak radiative corrections

We now turn to indirect searches for, or constraints on, physics beyond the Standard Model, from precision measurements at LEP and elsewhere that are sensitive to electroweak radiative corrections. These are sensitive within the Standard Model to the unknown masses of the top quark and the Higgs boson. There is good consistency between low-energy data and the high-energy data which are dominated by LEP, and a global analysis (Ellis & Fogli 1990) gives

$$m_t = 127^{+24}_{-30}\,\text{GeV} \tag{14a}$$

for $m_H = m_Z$, and

$$m_t = 122^{+41}_{-32}\,\text{GeV} \tag{14b}$$

if m_H is left free. The precision electroweak data even start to exhibit some sensitivity to the Higgs mass, and figure 6 shows the 68% confidence level contour in the (m_H, m_t) plane.

One immediate question is whether supersymmetry could alter significantly the preferred range of m_t. One expects the largest supersymmetric contributions to radiative corrections such as ρ to be those due to $\tilde{t}$ and $\tilde{b}$ loops. These would be as large as those due to t and b quarks if $m_{\tilde{q}} \ll m_W, m_t$, but we know from direct searches that such is not the case. Assuming that $m_{\tilde{q}} \gtrsim 150\,\text{GeV}$ as indicated by the CDF experiment, we find (Bilal *et al.* 1990) that for $\tan\beta \gtrsim 1.3$ the central value of m_t is reduced by at most 10 GeV, while the $1-\sigma$ upper limit is reduced by at most 20 GeV, and the $1-\sigma$ lower limit is unaltered. Figure 7 shows a typical comparison of the central values of $\sin^2\theta_W$ extracted from different classes of experiments with and without the inclusion of supersymmetric radiative corrections. We conclude that their effect is marginal, and the only effect they might have would be to reduce slightly the preferred range of m_t. Certainly, what we have as yet measured of radiative corrections does not constrain supersymmetry.

To discuss other possible extensions of the Standard Model, it is convenient (Peskin & Takeuchi 1990) to use a general approximate parametrization of radiative corrections in terms of two quantities

$$T \equiv \frac{1}{\alpha(m_Z)}\left(\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right), \tag{15a}$$

which parametrizes isospin violation, and

$$S \equiv \frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2\,\alpha(m_Z)/4\sin^2\theta_W} \approx \frac{\Pi_{ZZ}(m_x^2) - \Pi_{ZZ}(0)}{m_Z^2\,\alpha(m_Z)/4\sin^2\theta_W\cos^2\theta_W}, \tag{15b}$$

which parametrizes the variations with momenta in the vector boson propagators. They are related to more familiar quantities by

$$\rho = 1 + \alpha(m_Z)\,T \tag{16a}$$

and

$$\bar{\Delta}r = \alpha(M_Z)\,S/4\sin^2\theta_W\cos^2\theta_W - \alpha(m_Z)\,T. \tag{16b}$$

The usual electroweak observables can be expressed conveniently in terms of S and T, for example (Marciano & Rosner 1990):

$$M_W = 80.20 - 0.29 \cdot S + 0.45 \cdot T\,\text{GeV}, \tag{17a}$$

$$\Gamma_Z/\Gamma_Z^0 = 1 - 0.0038 \cdot S + 0.0105\,T, \tag{17b}$$

$$Q_W^{PV} = -73.20 - 0.8 \cdot S - 0.005\,T, \tag{17c}$$

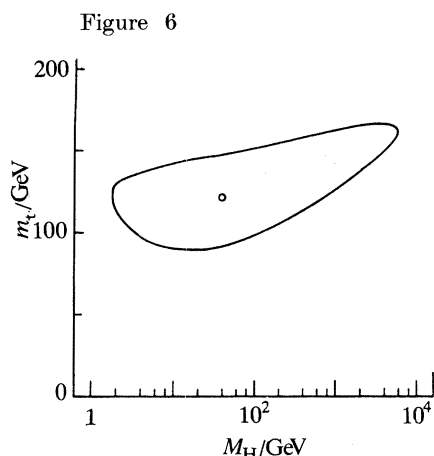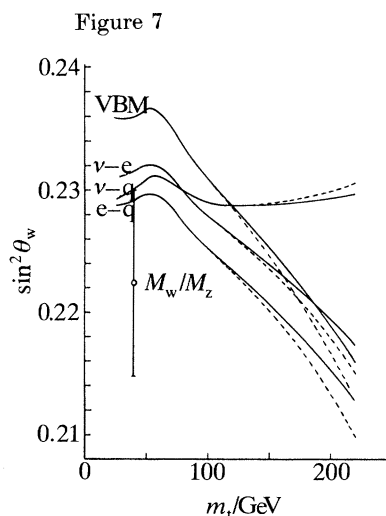Figure 7

Figure 6

Figure 6. Region of the $(m_\mathrm{H}, m_\mathrm{t})$ plane allowed by a global analysis of electroweak measurements (Ellis & Fogli 1990).

Figure 7. Central values of $\sin^2\theta_\mathrm{W}$ extracted from different electroweak sectors without (solid lines) and with (dashed lines) supersymmetric contributions to the radiative corrections (Bilal *et al.* 1990).

where the superscript zero denotes a tree-level quantity, and Q_W^PV is a measure of parity violation in caesium. The Standard Model contributions to S and T are

$$\Delta T \approx \frac{3}{16\sin^2\theta_\mathrm{W}}\left(\frac{m_\mathrm{t}^2-(140\,\mathrm{GeV})^2}{m_\mathrm{W}^2}\right)-\frac{3}{16\pi\cos^2\theta_\mathrm{W}}\ln\left(\frac{m_\mathrm{H}}{100\,\mathrm{GeV}}\right)^2 \tag{18a}$$

and

$$\Delta S \approx (1/6\pi)\ln(m_\mathrm{H}/100\,\mathrm{GeV}), \tag{18b}$$

where reference values $m_\mathrm{t} = 140\,\mathrm{GeV}$, $m_\mathrm{H} = 100\,\mathrm{GeV}$ have been adopted. Our general analysis (Ellis & Fogli 1990) of radiative corrections, which was previously analysed in terms of m_t and m_H, can now be reanalysed in terms of S and T:

$$-0.5 \leqslant T \leqslant 0.6, \quad -0.22 \leqslant S \leqslant 0.22, \tag{19}$$

at the 68% confidence level. Using the bound $m_\mathrm{H} > 50\,\mathrm{GeV}$ from direct searches (Green 1991), we infer that the contribution to S from beyond the Standard Model is bounded by

$$\Delta S_\mathrm{BSM} \leqslant 0.25. \tag{20}$$

This is bad news for technicolour models, in which there is a composite Higgs boson weighing *ca.* $1\,\mathrm{TeV}$ and a contribution (Peskin & Takeuchi 1990)

$$\Delta S_\mathrm{TC} \approx 2.0+0.4\,(N_\mathrm{T}-4) \tag{21}$$

for a model with one generation and N_T technicolours. The conventional one-generation technicolour model would therefore give $\Delta S_\mathrm{TD} \approx 2$, which is apparently excluded by the electroweak data (21). One can also use the bound (21) to constrain models with heavy fermion doublets, which give

$$\Delta S_\mathrm{HF} \approx (1/6\pi)N_\mathrm{HF}. \tag{22}$$

The data (21) apparently constrain the number of heavy fermion doublets: $N_\mathrm{HF} < 5$.

4. Extrapolation to higher energies

All field-theoretical couplings vary logarithmically with the energy scale at which they are measured, an effect which has been observed experimentally for the strong coupling. The asymptotic freedom of QCD can be extrapolated until the strong and electroweak couplings become comparable, and their unification within a simple gauge group: $G \supset SU(3) \times SU(2) \times U(1)$ is possible. Because the couplings only evolve logarithmically, this grand unification can only occur at an exponentially high-energy scale m_X:

$$m_X = \exp(O(1)/\alpha_{em}). \tag{23}$$

Baryon stability requires $m_X > 10^{14}$ GeV, and the neglect of gravity in this unification programme is consistent only if $m_X < M_P \approx 10^{19}$ GeV. The formula (23) yields m_X in this range only if (Ellis & Nanopoulos 1980)

$$\tfrac{1}{120} > \alpha_{em} > \tfrac{1}{170}. \tag{24}$$

The fact that the measured value of the fine structure constant sits comfortably within this range encourages us to pursue this grand unification philosophy. More precise calculations using just the particles in the minimal Standard Model would indicate that (Ellis *et al.* 1980)

$$m_X \approx (1 \text{ to } 2) \times 10^{15} \Lambda_{\overline{MS}}^4 \approx (1 \text{ to } 4) \times 10^{14} \text{ GeV}, \tag{25}$$

where $\Lambda_{\overline{MS}}^4$ is the QCD scale parameter Λ in the $\overline{MS}$ prescription with four light flavours. In this prescription, the gauge couplings become strictly equal at a renormalization scale which is related to m_X by a simple and known numerical factor (Hall, 1979). In other prescriptions, such as momentum space regularization, the couplings only become equal asymptotically, but the physics is of course the same. The estimate (25) of m_X is largely independent of one's specific GUT model, the simplest of which uses $G = SU(5)$ (Georgi & Glashow 1974).

In point of fact, as seen in figure 8*a*, the LEP data show (Amaldi *et al.* 1991) that the gauge couplings do not all become equal at the same energy m_X, if only the Standard Model particles are included in their evolution equations. Moreover, the value (25) of m_X would very likely lead to too short a proton lifetime. Both these problems are solved by supersymmetric GUTs.

The radiative corrections due to sparticles slow the rate of approach of the Standard Model couplings, and equality at some energy (Ellis 1990*a*)

$$m_X \approx 1 \times 10^{16} \text{ GeV} \tag{26}$$

was highly consistent with the LEP values of the strong coupling

$$\alpha_s(m_Z) = 0.115 \pm 0.016 \tag{27a}$$

and

$$\sin^2 \theta_W^{\overline{MS}}(m_Z) = 0.2329 \pm 0.0013 \tag{27b}$$

reported in the summer of 1990, as seen in figure 8*b*. Moreover, in many supersymmetric GUTs it is possible to relate the b quark and τ lepton masses, which are equal at the grand unification scale m_X (Chanowitz *et al.* 1977). When one calculates the ratio m_b/m_τ of physical fermion masses, and puts in the measured value $m_\tau = 1.78$ GeV, one calculates (Ellis *et al.* 1990*a*)

$$m_b = 5.2 \pm 0.3 \text{ GeV} \tag{28}$$

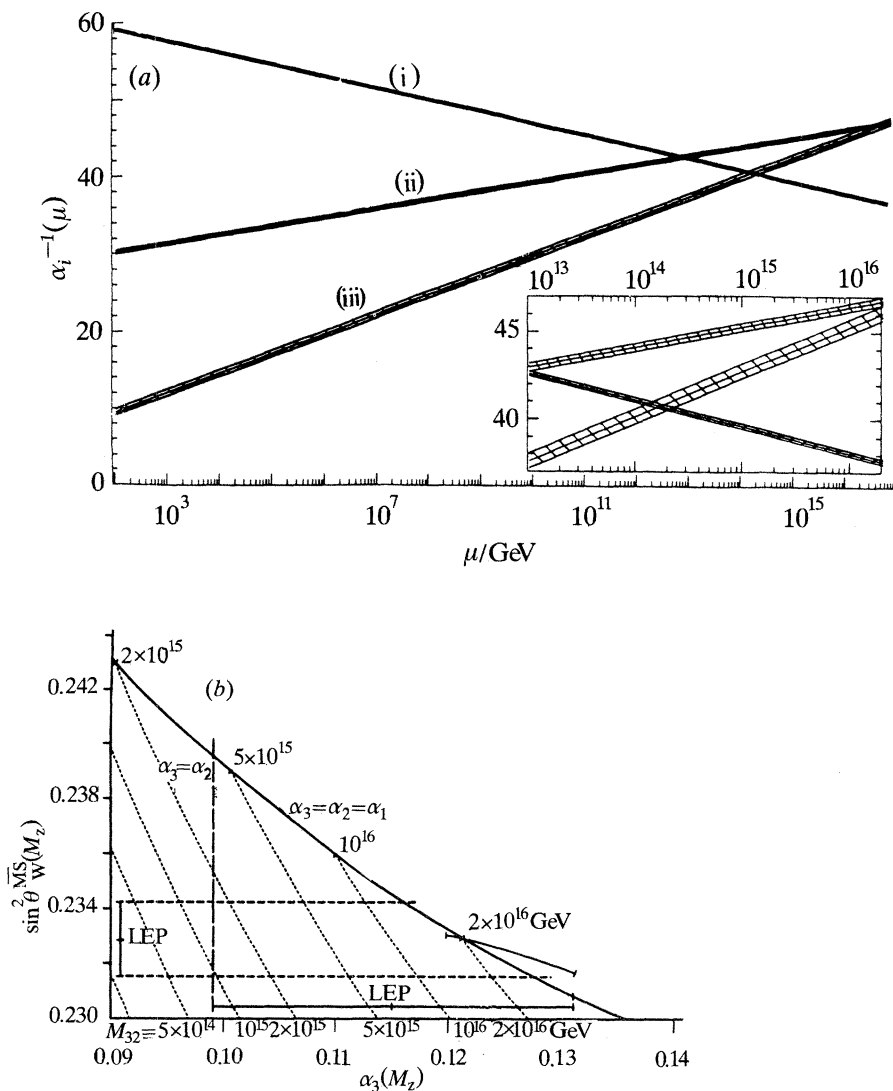as seen in figure 9 for the allowed ranges of $\alpha_3(m_Z)$ and m_t. This prediction agrees with

Figure 8. Extrapolation of the low-energy couplings measured at LEP are (a) inconsistent with minimal SU(5) grand unification (Amaldi *et al.* 1991), but (b) consistent with the minimal supersymmetric SU(5) GUT (Ellis *et al.* 1990a). (a) (i) $\alpha_1^{-1}(\mu)$, (ii) $\alpha_2^{-1}(\mu)$, (iii) $\alpha_3^{-1}(\mu)$.

the range $m_b = 5.0 \pm 0.2$ of masses required in potential model calculations, but holds if and only if there are just three matter generations: m_b would have been unacceptably high if there were four or more generations (Buras *et al.* 1978; Nanopoulos & Ross 1979, 1982*a, b*). The success of this prediction was used to predict that there were only three generations long before cosmological nucleosynthesis (Olive *et al.* 1990) and LEP converged on this number.

LEP data certainly strengthen the case for supersymmetric grand unification as compared with non-supersymmetric GUTs, but it may be premature to use them to set a strong upper bound on the possible threshold for introducing supersymmetry.
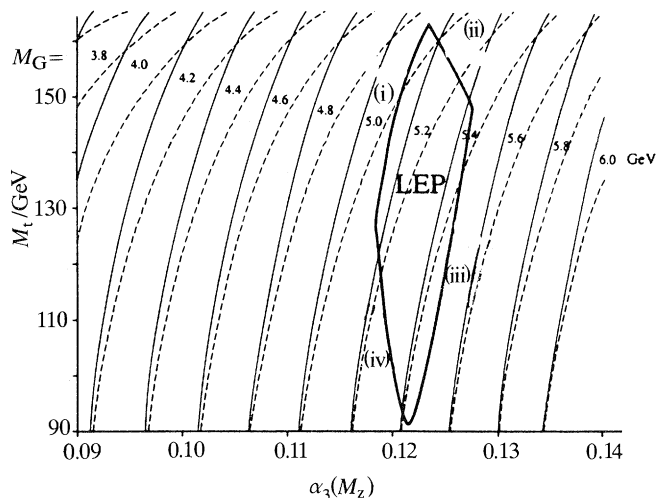
Figure 9. Contours of m_b calculated in minimal supersymmetric SU(5), assuming LEP values of α_3 and m_t for different values of $\tan\beta$ (solid and dashed lines) (Ellis *et al.* 1990*a*). (i) $(m_H)_{max}$, (ii) $(m_t)_{max}$, (iii) $(m_H)_{min}$, (iv) $(m_t)_{min}$.

5. Probing string

As was discussed in the introduction, string (Green *et al.* 1986) is the only serious candidate for a theory of everything (TOE). According to this theory, point-like 'elementary' particles are replaced by loops of string extended over the Planck length $l_P \approx 10^{-33}$ cm. Because they are extended objects, there are an infinite number of excited states with squared masses

$$m_n^2 \approx \text{`0'} + n \times O(m_P^2). \tag{29}$$

The 'lowest harmonics' with $n = 0$ are the particles with masses very much less than m_P which we see. The 'higher harmonics' with $n > 0$ have masses of order than 10^{19} GeV or more, and are not directly accessible to accelerator experiments.

It used to be thought that string theories could only be quantized consistently in a large number of space-time dimensions: 10 for supersymmetric string models or 26 for the bosonic string. Subsequently, it was realized that some of these extra dimensions could be compactified on distance scales *ca.* $l_P \approx 10^{-33}$ cm, leaving a residual four-dimensional effective low-energy theory (Candelas *et al.* 1985). Nowadays, we know that it is not necessary to use the language of compactification, and one can instead formulate string theories directly in four dimensions, replacing the other dimensions by a suitable combination of internal degrees of freedom, which can correspond to a large gauge symmetry.

There are many possible string models, many of which have non-unified gauge groups of the form $[SU(3)]^m \times [SU(2)]^n \times [U(1)]^p$ (Font *et al.* 1990). If one looks for a grand unified model with a simple group $G \supset SU(3)_C \times SU(2)_L$, there is one outstanding obstacle, namely that models built using the existing technology based on a level-one Kac–Moody algebra cannot contain adjoint Higgs representations. These are essential in almost all GUTs, being needed to break G down to $SU(3) \times SU(2) \times U(1)$. A possible way out of this dilemma is provided by the only known GUT that does not require adjoint Higgs fields, namely flipped $SU(5) \times U(1)$
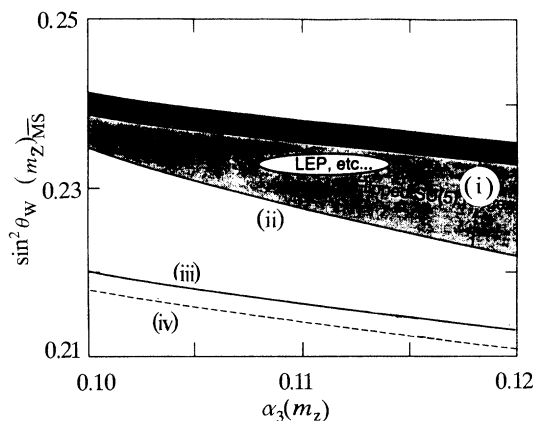
Figure 10. Confrontation of more refined data on α_3 and $\sin^2\theta_W$ with the predictions of non-supersymmetric SU(5), supersymmetric SU(5), and flipped SU(5) × U(1) (Ellis *et al.* 1990*b*). (i) Flipped SU(5) range; (ii) τ_{proton}; (iii) minimal SU(5):1 Higgs doublet; (iv) 2 Higgs doublets.

(Antoniadis *et al.* 1987, 1988*a, b*, 1989), that can be broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by $\underline{10}$ and $\underline{\overline{10}}$ representations of Higgses. In this model, the standard GUT unification condition $\alpha_3 = \alpha_2 = \alpha_1$ at m_X is relaxed to become $\alpha_3 = \alpha_2$ at $\mathcal{M}_{\text{GUT}} \leqslant \mathcal{M}_{\text{SU}}$, where $\alpha_5 = \alpha_1$. Therefore, in this model $\alpha_3 = \alpha_2 \geqslant \alpha_1$ at $\mathcal{M}_{\text{GUT}}$, and $\sin^2\theta_W$ can be lower than in standard SU(5).

Since last summer, there have been new determinations of $\alpha_3(m_Z)$ from LEP (de Boer 1990):

$$\alpha_3(m_Z) = 0.107 \pm 0.007 \tag{30a}$$

and from deep inelastic scattering (Martin *et al.* 1990):

$$\alpha_3(m_Z) = 0.112 \pm 0.003. \tag{30b}$$

We have tested the compatibility of these values and the LEP value (27*b*) of $\sin^2\theta_W$ with minimal supersymmetric SU(5) unification and flipped SU(5) × U(1) (Ellis *et al.* 1990*b*). We assume the pattern (5) of supersymmetry breaking, and allow the sparticle masses to vary between 40 GeV and 1 TeV. As seen in figure 10, the data are far from minimal non-supersymmetric SU(5) unification, and very close to the range allowed by minimal supersymmetric SU(5). It remains to be seen whether the apparent $1 - \sigma$ discrepancy will turn out to be significant. The data lie well within the range allowed by flipped SU(5) × U(1) and proton stability.

It is interesting to compare the range of unification scales allowed by LEP with the string unification scale calculated (Kaplunovsky 1988; Dixon *et al.* 1990) for a large class of string models:

$$\mathcal{M}_{\text{SU}} = (1/4\pi\sqrt{\alpha'})\,\hat{\mathcal{M}}, \tag{31a}$$

where $\alpha' = g^2/8\pi m_P^2$ and

$$\hat{\mathcal{M}} = \prod_{\alpha=1}^{3} [\sqrt{(\operatorname{Im} T_\alpha)}|\,\eta(T_\alpha)|^2]^{\lambda_\alpha} : \sum_{\alpha=1}^{3} \lambda_\alpha = -1, \tag{31b}$$

where the T_α correspond to the so-called moduli of compactification, and η is the Dedekind function. Numerically, (31) yields

$$M_{\text{SU}} = 1.93 \times 10^{17}\,\text{GeV} \times g \times \hat{M}. \tag{32}$$

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LEP allows values of M_{SU} corresponding to (Ellis *et al.* 1990 *a*)

$$\hat{M} = \tfrac{1}{3} \text{ to } \tfrac{1}{15}, \tag{33}$$

which is compatible with plausible values of the moduli T_α in (31*b*).

Finally, we note that one can predict Yukawa couplings in many four-dimensional string models:

$$\lambda = O(1) \times g \tag{34}$$

when λ and g are measured at the string unification scale. Specifically, in the flipped $\mathrm{SU}(5) \times \mathrm{U}(1)$ superstring GUT (Antoniadis *et al.* 1989) one has

$$\lambda = 2^{\frac{1}{2}} g \tag{35}$$

for all non-zero Yukawa couplings λ. However, most Yukawa couplings vanish, and the Yukawa coupling of the physical t quark contains an unknown mixing factor

$$\lambda_{\mathrm{t}} = 2^{\frac{1}{2}} g \cos \theta_{\mathrm{t}}. \tag{36}$$

This gives a range of possible top quark Yukawa couplings at the string scale that corresponds to a physical top quark mass $m_{\mathrm{t}} \lesssim 190 \text{ GeV}$ (Antoniadis *et al.* 1989). Certainly the range (14) indicated by the analysis of electroweak radiative corrections (Ellis & Fogli 1990) is compatible with this string range.

The puzzle is not why is the top quark so heavy, but rather why are the other quarks and leptons so light? The value $m_{\mathrm{t}} = 123 \text{ GeV}$ corresponds to a Yukawa coupling $\lambda_{\mathrm{t}} \approx 1/\sqrt{2}$, which seems a reasonable value, whereas $m_{\mathrm{e}} = 0.51 \text{ MeV}$ corresponds to $\lambda_{\mathrm{e}} \approx 1.5 \times 10^{-6}$. Such a small Yukawa coupling is technically natural, in the sense that radiative corrections are under control, but why is it so small? Perhaps the answer will be presented at a future Discussion Meeting?

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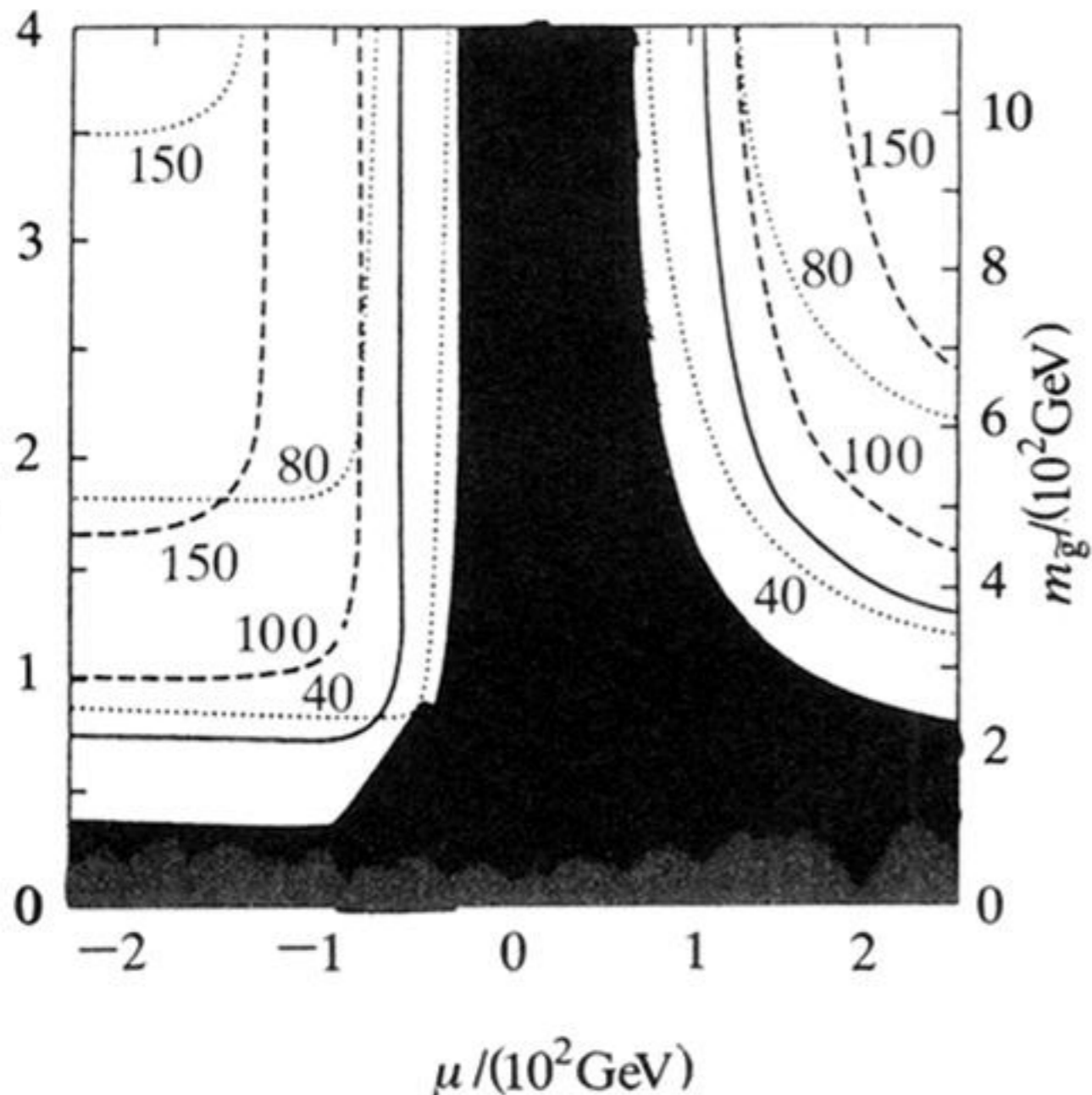
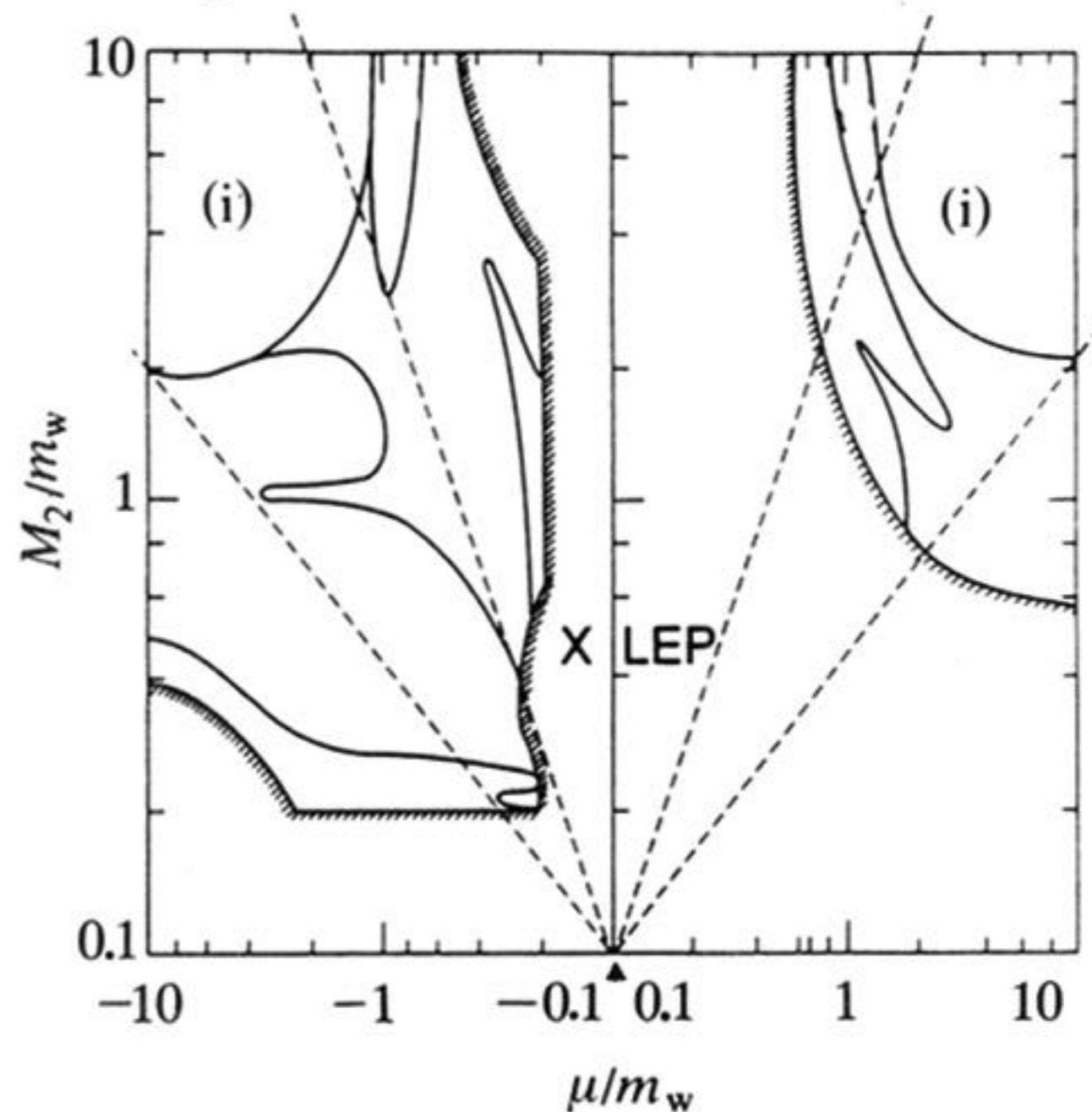Witten, E. 1981 *Nucl. Phys.* B **188**, 513.

Figure 2

Figure 3

Figure 2. Region of the H_1, H_2 mixing parameter μ and the supersymmetry-breaking parameter $m_{\frac{1}{2}}$ excluded by LEP and CDF data for $\tan\beta = 2$ (shaded area) (Ridolfi *et al.* 1990).

Figure 3. Regions of the $(\mu, m_{\frac{1}{2}})$ plane in which the lightest supersymmetric particle (LSP) may have the closure density $\Omega = 1$ for some choice of the parameters $(\tan\beta, m_A, m_{\tilde{q}})$ (Ellis *et al.* 1990d); region (i), $m_{LSP} > m_W$.